

**UNITED STATES PATENT APPLICATION**

*of*

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**RANDOMIZER SYSTEMS FOR PRODUCING MULTIPLE-SYMBOL  
RANDOMIZING SEQUENCES**

Patent Application

# RANDOMIZER SYSTEMS FOR PRODUCING MULTIPLE- SYMBOL RANDOMIZING SEQUENCES

## BACKGROUND OF THE INVENTION

A randomizer sequence is a long, non-repeating sequence of symbols or bits that  
5 combine with a code word in order to randomize the code word symbols. One reason to  
randomize the symbols of the code word is to eliminate repeated patterns of symbols,  
such as patterns of all zero symbols, that may not always be accurately demodulated.  
The code words of interest are error correction code words that are produced by encoding  
data in accordance with an error correction code (ECC) over a Galois Field  $GF(2^m)$ .  
10 Over  $GF(2^m)$ , the sequence and the code word are combined by XOR'ing.

Before decoding, the randomizer sequence is removed from the randomized code  
word by combining the same sequence with the code word. The ECC code word can  
then be decoded in a conventional manner and errors in the code word data symbols  
corrected, as appropriate.

15 Randomizer circuits for producing multiple-bit non-repeating sequences are  
known. One such circuit is a binary maximal length linear feedback shift register, such  
as an m-bit shift register that produces a random sequence up to  $2^m-1$  bits. The bit  
sequence, however, is not generally long enough to combine with all of the bits of the  
ECC code word. Accordingly, the  $2^m-1$  bit sequence, or pattern, must be used multiple  
20 times in the same code word, and the randomizing of the code word symbols may be  
adversely affected.

## SUMMARY OF THE INVENTION

The invention is a system that operates in accordance with one or more multiplier  
constants that are primitive elements of  $GF(2^m)$  to produce multiple-symbol non-

repeating randomizer sequences that are long enough to combine with all the symbols of ECC code words that are encoded over  $GF(2^m)$ . The randomizer sequence is thus used once in a given code word.

The system may, without rewiring, produce a number of different multiple-  
5 symbol randomizer sequences, such that a key may be needed to select the particular sequence used with a given code word. Accordingly, the system can be used also to encrypt the ECC code word. Further, the multiplier constants may be selected such that each randomizer sequence produced by the system is separated from every valid ECC code word by a predetermined minimum distance, as discussed in more detail below.  
10 The system can thus be used to provide mis-synchronization detection.

More specifically, the system includes a circuit that is set up in accordance with a polynomial with at least one coefficient, or multiplier constant, that is a primitive element of  $GF(2^m)$ , and produces a non-repeating sequence that includes  $2^m-1$  or more m-bit symbols. The particular sequence produced by the circuit is determined by the primitive  
15 element selected as the multiplier constant and the selected initial state of one or more registers in the circuit. The initial state is selected such that at least one of the registers contains a non-zero element of  $GF(2^m)$ , and thus,  $2^m-1$  different sequences may be produced by, for example, selecting a different initial state for the given register. If the primitive multiplier constant is also selected from, for example, a set of "p" possible  
20 multiplier constants, the system produces  $p*(2^m-1)$  different sequences. If additional ones of the initial register states and/or the multiplier constants are selectable, the system may produce even greater numbers of sequences. Accordingly, the system may be used to encrypt the ECC code word by associating the code word with a particular initial state and/or multiplier constant. An unauthorized user would then have to try many possible  
25 multiple-symbol sequences, in order to remove the randomizer sequence from the randomized code word and reproduce the ECC code word.

The inventive system provides benefits that are not provided by the known prior randomizer systems. These prior systems produce relatively short random sequences that must be repeated in the code word and are thus not appropriate for encryption of the  
30 entire code word. Further, the prior systems produce sequences that are not necessarily

optimized for mis-synchronization detection, that is, sequences that are not predetermined minimum distances away from the valid ECC code words. Accordingly, the known prior randomizing systems do not provide as robust mis-synchronization detection

## BRIEF DESCRIPTION OF THE DRAWINGS

The invention description below refers to the accompanying drawings, of which:

Fig. 1 is a functional block diagram of an encoding system constructed in accordance with the invention;

Fig. 2 is a functional block diagram of a decoding system constructed in accordance with the invention;

Fig. 3 is a more detailed functional block diagram of a randomizer circuit included in the systems of Figs. 1 and 2;

Fig. 4 is a functional block diagram of an alternative randomizer circuit; and

Figs. 5 and 6 are functional block diagrams of further randomizer circuits.

## DETAILED DESCRIPTION OF AN ILLUSTRATIVE EMBODIMENT

Referring to Fig. 1, an encoder 2 encodes data in a known manner in accordance with a BCH code, for example, a distance  $d$  Reed-Solomon code over  $GF(2^m)$ , to produce an ECC code word. At the same time a randomizer circuit 10 produces a multiple-symbol, non-repeating randomizer sequence. The symbols of the randomizer sequence are XOR'd to the corresponding symbols of the ECC code word in XOR gates 4, to produce a randomized code word for recording or transmitting.

Referring now to Fig. 2, when the randomized code word is later retrieved or received, a decoder 8 removes the randomizer sequence from the code word in XOR gates 4, before the code word is decoded by a conventional decoding subsystem 6. Accordingly, the randomizer circuit 10 used for decoding must generate the same randomizer sequence produced by the encoder 2 (Fig. 1). If necessary, the decoder 8 uses information in a key 9 to set the randomizer circuit 10 to the appropriate initial state, as discussed below. The key 9 is provided to the decoder 8 in a conventional manner.

Referring now to Fig. 3, a randomizer circuit 10 that is set up in accordance with a degree-one polynomial includes a multiplier 12, and a register 14 that is initially set to a non-zero element of  $GF(2^m)$ . The randomizer circuit depicted in the drawing generates a multiple-symbol randomizer sequence using the polynomial  $x + \alpha^k$ , where  $\alpha^k$  is a primitive element of  $GF(2^m)$ . The sequence produced by the circuit is:

$$S_k = R\alpha^k, R(\alpha^k)^2, R(\alpha^k)^3 \dots R(\alpha^k)^i$$

which is non-repeating for  $i \leq 2^m - 1$ . By selecting R as different ones of the  $2^m - 1$  non-zero symbols of  $GF(2^m)$ , the system may produce  $2^m - 1$  different, non-repeating randomizer sequences, with each sequence including up to  $2^m - 1$  m-bit symbols. The system may store particular values for R or may generate them by, for example, raising  $\alpha$  to a selected power.

The system must use the selected value of R to reproduce the same sequence for decoding. As appropriate, the key 9 specifies or points to the selected value of R or, for example, the selected power to which  $\alpha$  is raised to produce R.

For more robust encryption, the particular multiplier constant used to produce the randomizer sequence for a given code word may be selected from a set of "p" values. A general purpose multiplier (not shown) may then be used in the circuit 10 in place of the constant multiplier 12. Alternatively, as depicted in Fig. 4, p multipliers 12 may be included in the circuit, with a switch 16 selecting the particular multiplier to be used to produce the randomizer sequence for a given code word. The key supplied to the decoder must then include information that specifies both R and  $\alpha^k$ . If the key is not provided, a user must test a possible  $p \cdot (2^m - 1)$  sequences in order to remove the randomizer sequence from the randomized code word.

To provide mis-synchronization detection, the multiplier constant  $\alpha^k$  may be selected to produce a randomizer sequence  $S_k$  that is a predetermined minimum distance away from every code word of the distance d Reed-Solomon code such that a mis-synchronized error results in a decoded code word that contains more errors than the ECC can correct.

As discussed above, the multiplier constant may be selected from a set of  $p$  values. For mis-synchronization detection, the values in the set must each produce sequences that satisfy the predetermined minimum distance requirements. Generally, if  $\alpha^k$  produces a randomizer sequence that meets the predetermined minimum distance requirements,  $\alpha^{2k}, \alpha^{4k}, \alpha^{8k}, \dots, \alpha^{2^m k}$  also produces such a randomizer sequence.

Accordingly, the  $p$  multiplier constants can be determined from  $\frac{p}{m}$  tested values of  $\alpha^k$ .

Referring now to Fig. 5, circuits that use polynomials with degrees greater than one may also be used to produce longer multiple-symbol, non-repeating randomizer sequences. The higher-degree randomizer circuits produce sequences of up to  $(2^m)^y - 1$  symbols, where  $y$  is the degree of the polynomial. These circuits may thus be used in systems in which the ECC is a primitive BCH code of degree  $y$ .

The drawing depicts a randomizer circuit 100 that is set up in accordance with a degree-two polynomial. The randomizer circuit 100 shown in the drawing includes two multipliers 102<sub>1</sub> and 102<sub>2</sub> and produces the randomizer sequence in accordance with the polynomial  $x^2 + \beta x + \gamma$ , where  $\beta$  and  $\gamma$  are primitive elements of  $GF(2^m)$ . For any pair of primitive elements  $\beta$  and  $\gamma$  the randomizer sequence produced by the randomizer circuit 100 is specified by  $R_1$  and  $R_2$ , which may be, respectively, any elements of  $GF(2^m)$ , where at least one of  $R_1$  and  $R_2$  is non-zero. There are thus many more possibilities for the randomizer sequences with this system since the values of  $\beta, \gamma, R_1$  and  $R_2$  may each be selected and the randomizer circuit 100 may thus provide more robust encryption than the system depicted in Fig. 1. The key that is supplied for decoding must specify the selected values, so that randomizer system can be set to the appropriate initial state to reproduce the randomizing sequence.

For mis-synchronization detection, the values of the multiplier constants and at least one of the values for the register are preferably selected together, to ensure that associate randomizer sequence satisfies the predetermined minimum distance requirements. For each selection, there are  $2^m - 1$  possibilities for the associated value of the remaining register, and thus,  $2^m - 1$  possible sequences.

Fig. 6 depicts an alternative degree-two randomizer circuit 110. Again, the circuit produces multiple-symbol randomizer sequences for selected values of  $\beta, \gamma, R_1$  and/or  $R_2$ .

The foregoing description has been limited to specific embodiments of this invention. It will be apparent, however, that variations and modifications may be made  
5 to the invention, with the attainment of some or all of its advantages. Therefore, it is the object of the appended claims to cover all such variations and modifications as come within the true spirit and scope of the invention.

What is claimed is:

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